Asymptotically Exact Solution for Superconductivity near Ferromagnetic Criticality

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(February 2, 2008)

We analyze an asymptotically exact solution for the transition temperature of p-wave superconductivity near ferromagnetic criticality on the basis of three-dimensional electron systems in which scattering processes are dominated by exchange interactions with small momentum transfers. Taking into account all Feynman diagrams in the gap equation, we show that vertex corrections neglected in the conventional Eliashberg formalism enhance the dynamical retardation effect of the pairing interaction, and increase the superconducting transition temperature significantly, though they just give subleading corrections to properties of the normal state.

The recent discovery of ferromagnetic superconductors such as UGe_2, URhGe, and $ZrZn_2$ has brought about renewed interest in superconductivity (SC) in the vicinity of ferromagnetic criticality (FC) [1–3]. The proximity to FC implies a possible spin-fluctuation-mediated Cooper pairing between parallel spins [4–11]. For more than 20 years, the transition temperatures T_c of paramagnon-mediated SC have been evaluated by solving the Eliashberg equation in which vertex corrections are neglected [4,5], though it has been recognized by researchers that the vertex corrections are important in the paramagnon theory [12]. A few studies based on finite-order perturbative calculations have revealed that a certain type of vertex correction increases T_c [13]. Nevertheless, it is highly desirable to elucidate the role of vertex corrections played in spin-fluctuation-mediated SC by non-perturbative approaches. To obtain a better physical insight into this issue, in the present paper, we analyze an electronic system with a pure repulsive interaction which is *exactly* proved to exhibit ferromagnetic critical behaviors and also the transition to spin-triplet SC. The superconducting transition temperature T_c is determined by including full-order vertex corrections, and compared with that obtained by neglecting vertex corrections. We will show that although vertex corrections to the single-particle normal self-energy are negligible near FC, T_c is substantially increased by vertex corrections to the gap equation.

The model considered in the present paper is a three-dimensional (3D) electronic system in which the ferromagnetic spin exchange interaction is dominated by forward scattering with small momentum transfers. The model Hamiltonian is of the form,

$$H = \sum_k E_k c^\dagger_{\sigma k} c_{\sigma k} - \sum_q V(q) \boldsymbol{S}(q) \cdot \boldsymbol{S}(-q), \tag{1}$$

where $\boldsymbol{S}(q) = \sum_k c^\dagger_{\alpha k+q} \boldsymbol{\sigma}_{\alpha\beta} c_{\beta k}$, $V(q) = U\Theta(q_c - |q_\perp|)$ with $\Theta(x) = 1$ for $x \geq 0$ and 0 for $x < 0$, and U is positive. In this system, momentum transfers in the direction tangent to the Fermi surface $q_\perp$ are restricted to a small range $|q_\perp| < q_c$, though momentum transfers in the direction normal to the Fermi surface are not. The system can be regarded as a low-energy effective theory for electron systems near FC.

Since, in 3D itinerant electron systems close to FC, the Gaussian fixed point is stable [14,15], a RPA-like treatment of the ferromagnetic spin fluctuation is valid for our system. Moreover, because of the reason explained below, the spin correlators of model (1) are almost exactly given by RPA-like expressions *with unrenormalized parameters*. It is noted that model (1) is a variant of the fermion systems with strong forward scattering that has been studied extensively by several authors [16–18]. In these systems, the expansion with respect to a small parameter $q_c/(2k_F)$ enables us to treat electron-electron interaction in all orders in terms of the coupling strength U. Since the low-energy properties of the systems are mainly determined by scattering processes with small momentum transfers, the results obtained up to leading order in $q_c/(2k_F)$ are *asymptotically exact* at sufficiently low temperatures. To implement this analysis, we exploit the remarkable theorem proved by Metzner et al. and Kopietz et al.; for leading order in $q_c/(2k_F)$, fermionic loop diagrams with more than two insertions cancel each other in strong forward scattering fermion systems that do not involve spin flip processes [16,17]. The proof given by these authors for this loop cancellation theorem can be straightforwardly generalized to the case of spin exchange interactions in the paramagnetic state; i.e. *the loop cancellation theorem holds, even when spin flip occurs at vertices.* However, we should keep in mind the following

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limitations of the theorem. In the ferromagnetic state, the cancellation among loop diagrams involving spin flip is not complete, as is easily checked by considering a loop with three legs involving spin exchange processes. Thus in the following we mainly concentrate on the paramagnetic state. A more serious restriction of our argument is that the theorem is not applicable to the superconducting state below T_c, because loop diagrams involving the condensation of Cooper pairs do not cancel [19]. Despite these limitations, the theorem is still useful in analyzing the transition temperature T_c, which is determined by the linearized gap equation as shown below.

The loop cancellation theorem ensures that the effective interaction between electrons with parallel spins for model (1) is the RPA-like form,

$$D_q(\omega_n) = \frac{-V^2(q)\chi_0(q,\omega_n)}{1-V(q)\chi_0(q,\omega_n)} \approx \frac{-3U}{t+(\frac{q}{2k_F})^2+\frac{3\pi|\omega_n|}{2vq}}. \tag{2}$$

Here $\chi_0(q,\omega_n) = -T\sum_{k,m} G^0_k(\varepsilon_m)G^0_{k+q}(\varepsilon_m+\omega_n)$ with $G^0_k(\varepsilon_m) = 1/(i\varepsilon_m - E_k)$, and $t = 3[1/(UN(0))-1]$, $N(0) = k_F^3/(\pi^2 E_F)$. v is the Fermi velocity. Also, the effective interaction which involves spin exchange $D^{(ex)}_q(\omega)$ is given by $D^{(ex)}_q(\omega) = 2D_q(\omega)$ because of the spin rotational symmetry. In a similar manner, we can express exact spin correlation functions as RPA forms with unrenormalized parameters. Thus the transition to a ferromagnetic state is completely determined by the Stoner condition. In the following, we apply our analysis only to the paramagnetic case $UN(0) < 1$.

In general, RPA expressions with unrenormalized parameters are exact in the case where interactions between fluctuations of order parameters are infinitely long-ranged, and thus all self-energy corrections vanish [20,21]. We would like to emphasize that, in our system, the absence of self-energy corrections in eq. (2) is not due to this reason, but caused by loop cancellations for finite range interactions, and that the single-particle self-energy does not vanish by itself in model (1).

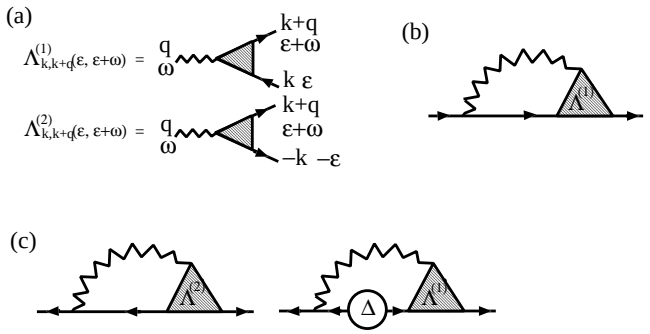

FIG. 1. (a) Feynman diagrams of three-point vertices. $\Lambda^{(1)}$ is the normal part. $\Lambda^{(2)}$ is the anomalous part. (b) Diagram of the normal self-energy. The wavy line indicates the effective interaction. (c) Diagrams of the anomalous self-energy.

For the exact analysis of the superconducting transition, we calculate three-point vertices including both normal and anomalous parts, whose diagrammatic expressions are shown in Fig. 1(a). As shown by Metzner et al., in systems with strong forward scattering, the irreducible current vertex $\mathbf{\Lambda}_{k,k+q}(\varepsilon,\varepsilon+\omega)$ is related to the irreducible density vertex $\Lambda_{k,k+q}(\varepsilon,\varepsilon+\omega)$ as $\mathbf{\Lambda}_{k,k+q}(\varepsilon,\varepsilon+\omega) \approx \boldsymbol{v}_{k+q/2}\Lambda_{k,k+q}(\varepsilon,\varepsilon+\omega)$, because of the velocity conservation [16]. Then, we obtain the asymptotic Ward identity for the normal part of the density three-point vertex,

$$\Lambda^{(1)}_{k,k+q}(\varepsilon,\varepsilon+\omega) = \frac{G^{-1}_{k+q}(\varepsilon+\omega) - G^{-1}_k(\varepsilon)}{i\omega - \boldsymbol{q}\cdot\boldsymbol{v}_{k+q/2}}, \tag{3}$$

where $G_k(\varepsilon)$ is the exact single-particle Green's function. The anomalous part of the density three-point vertex also satisfies the generalized asymptotic Ward identity,

$$\Lambda^{(2)}_{k,k+q}(\varepsilon,\varepsilon+\omega) = -\frac{\Delta_k(-\varepsilon)+\Delta_{k+q}(\varepsilon+\omega)}{i\omega - \boldsymbol{q}\cdot\boldsymbol{v}_{k+q/2}}. \tag{4}$$

Here $\Delta_k(\varepsilon)$ is the anomalous self-energy, which emerges as a result of the condensation of Cooper pairs.

Using the Ward identities (3) and (4), and eq. (2), we can write down the exact normal self-energy $\Sigma_k(\varepsilon)$ expressed diagrammatically in Fig. 1(b),

$$\Sigma_k(\varepsilon) = -g\sum_{k',\varepsilon'} D_{k-k'}(\varepsilon-\varepsilon')G_{k'}(\varepsilon')\Lambda^{(1)}_{k,k'}(\varepsilon,\varepsilon')$$
$$= g\sum_{k',\varepsilon'} \frac{D_{k-k'}(\varepsilon-\varepsilon')G_{k'}(\varepsilon')G_k^{-1}(\varepsilon)}{i(\varepsilon'-\varepsilon)-(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}_{(k+k')/2}} + gC, \tag{5}$$
$$C = -\sum_{k'\varepsilon'} \frac{D_{k-k'}(\varepsilon-\varepsilon')}{i(\varepsilon'-\varepsilon)-(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}_{(k+k')/2}}, \tag{6}$$

where the prefactor $g = 3$ stems from the SU(2) spin rotational symmetry, and the exact single-particle Green's function is $G_k(\varepsilon) = [Z_k(\varepsilon)i\varepsilon - E_k]^{-1}$ with $Z_k(\varepsilon) = 1-\Sigma_k(\varepsilon)/(i\varepsilon)$. The anomalous self-energy $\Delta_k(\varepsilon)$ at $T = T_c$ can be linearized in terms of the gap function $\Delta_k(\varepsilon)/Z_k(\varepsilon)$. The exact linearized gap equation expressed diagrammatically in Fig. 1(c) is then written as,

$$\Delta_k(\varepsilon) = \sum_{k',\varepsilon'} G_{-k'}(-\varepsilon')D_{k-k'}(\varepsilon-\varepsilon')\Lambda^{(2)}_{k,k'}(\varepsilon,\varepsilon')$$
$$+\sum_{k',\varepsilon'} \Delta_{k'}(\varepsilon')G_{k'}(\varepsilon')G_{-k'}(-\varepsilon')D_{k-k'}(\varepsilon-\varepsilon')\Lambda^{(1)}_{k,k'}(\varepsilon,\varepsilon')$$
$$= 2\sum_{k',\varepsilon'} \Delta_{k'}(\varepsilon')G_{k'}(\varepsilon')G_{-k'}(-\varepsilon')D_{k-k'}(\varepsilon-\varepsilon')\Lambda^{(1)}_{k,k'}(\varepsilon,\varepsilon')$$
$$+\Delta_k(\varepsilon)\sum_{k',\varepsilon'} \frac{D_{k-k'}(\varepsilon-\varepsilon')G_k(-\varepsilon')}{i(\varepsilon'-\varepsilon)-(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}_{(k+k')/2}}$$
$$-\sum_{k',\varepsilon'} \frac{\Delta_{k'}(\varepsilon')D_{k-k'}(\varepsilon-\varepsilon')G_{k'}(\varepsilon')G_{k'}(-\varepsilon')G_k^{-1}(\varepsilon)}{i(\varepsilon'-\varepsilon)-(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}_{(k+k')/2}}. \tag{7}$$

Note that the above equations are *exact* in the sense that full-order Feynman diagrams are taken into account, going beyond the conventional Eliashberg equations. In eqs. (6) and (7), the effects of superconducting fluctuations, which generally decrease T_c and give rise to pseudogap phenomena [22,21], do not exist, because the loop cancellation suppresses their contributions. Therefore, T_c is exactly determined by competition between the pairing interaction in (7) and the pair-breaking effect caused by the normal self-energy (6). Since the gap equation should be solved in the vicinity of the Fermi surface at low temperatures, $G_k^{-1}(\varepsilon)$ in the numerator of the last term of eq. (7) suppresses this term by a factor $\sim T/E_F$ compared to the other terms. Thus we drop it in the following.

To carry out a more explicit analysis, we consider an isotropic system with a spherical Fermi surface, and linearize the dispersion relation as $E_k = vk$. Then, we can easily show that the constant C in eq. (6) is equal to zero. Also, in this case, $(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{v}_{(k+k')/2} \approx (k'_r - k_r)v + 2k_F v\sin^2(\theta/2)$, where k_r is the component of momentum normal to the Fermi surface, and θ is the angle between $\boldsymbol{k}'$ and $\boldsymbol{k}$. This approximation is adopted in the denominators of the vertex functions in eqs. (6) and (7). Note that we retain the term quadratic in the transverse momentum transfers, $\sin^2(\theta/2)$, because, as was noted by Metzner et al. [16], this term is necessary to eliminate an artificial singularity in the vertex functions which destroys the Fermi liquid state. In the vicinity of FC, since low-energy fluctuations are dominant, we can neglect the momentum dependence of the self-energy, $Z_k(\varepsilon) \to Z(\varepsilon)$. We also expand Δ_k into the l-th angular momentum components Δ_l. Then, integrating over $E_{k'}$, we recast eqs. (6) and (7) into

$$[1-Z(\varepsilon_n)]\varepsilon_n = -gT\sum_m \int_0^{x_c} dx x\tilde{D}_0(x,\varepsilon_n-\varepsilon_m)$$
$$\times\left[\mathrm{sign}\varepsilon_m + \frac{i(\Sigma(\varepsilon_n)-\Sigma(\varepsilon_m))(\mathrm{sign}\varepsilon_m - \mathrm{sign}(\varepsilon_m-\varepsilon_n))}{|Z(\varepsilon_m)\varepsilon_m| + |\varepsilon_m-\varepsilon_n| + 2ik_F v x^2\mathrm{sign}(\varepsilon_m-\varepsilon_n)}\right], \tag{8}$$

$$[1-(2Z(\varepsilon_n))^{-1}]\Delta_l(\varepsilon_n) = T\sum_m \int_0^{x_c} dx x\tilde{D}_l(x,\varepsilon_n-\varepsilon_m)\frac{\Delta_l(\varepsilon_m)}{|Z(\varepsilon_m)\varepsilon_m|}$$
$$\times\left[1 + \frac{i(\Sigma(\varepsilon_n)-\Sigma(\varepsilon_m))\mathrm{sign}(\varepsilon_n-\varepsilon_m)}{|Z(\varepsilon_m)\varepsilon_m| + |\varepsilon_m-\varepsilon_n| + 2ik_F v x^2\mathrm{sign}(\varepsilon_m-\varepsilon_n)}\right], \tag{9}$$

$$\tilde{D}_l(x,\varepsilon-\varepsilon') = \frac{3\pi}{2}\frac{N(0)UP_l(1-2x^2)}{S(\varepsilon-\varepsilon')(S(\varepsilon-\varepsilon') + \frac{|Z(\varepsilon')\varepsilon'|}{2k_F v})}, \tag{10}$$

where $S^2(\omega) = t + x^2 + 3\pi|\omega|/(4vk_F x)$, $P_l(x)$ is the Legendre polynomial, and x_c is a cutoff which restricts the integral over the transverse momentum within the range $q_\perp < q_c$.

We see from eq. (8) that the vertex corrections to the normal self-energy contribute only in low-energy regions, because of the factor, $\mathrm{sign}\varepsilon_m - \mathrm{sign}(\varepsilon_m - \varepsilon_n)$, yielding only subleading corrections compared to the first term of the right-hand side of eq. (8). Thus, the low temperature behaviors of the normal self-energy are dominated by a one-loop diagram without vertex corrections. To check this more explicitly, we analytically continue to real frequencies, and obtain the low temperature behaviors of the normal self-energy. For $T < T^* = t^{3/2}E_F/(1.5\pi^2)$, the imaginary part of the self-energy shows the Fermi liquid (FL) behavior,

$$\mathrm{Im}\Sigma_k^R(0) = -\frac{27k_F^3U(\pi T)^2}{32E_F t^{3/2}} + O((T/E_F)^2\log(t)/t). \tag{11}$$

Here the second term is the subleading contribution from the vertex corrections. Accordingly, the mass renormalization factor is a finite constant,

$$\lambda = -\frac{\partial\Sigma_k^R(0)}{\partial\varepsilon} = \frac{3k_F^3U}{\pi^2E_F}\log(\frac{1+t}{t}) + O((T/E_F)^2). \tag{12}$$

For $T > T^*$, strong spin fluctuations give rise to non-Fermi liquid (NFL) behaviors,

$$\mathrm{Im}\Sigma_k^R(0) = -\frac{9k_F^3U}{2\pi}\frac{T}{E_F}\log(T/T^*) + O((T/E_F)^2), \tag{13}$$

$$\lambda = \frac{3k_F^3U}{\pi^2E_F}\log(\frac{1.14E_F}{T}) + O((T/E_F)^2). \tag{14}$$

The leading temperature dependences of the single-particle properties coincide with those obtained by lowest order (self-consistent or non-self-consistent) calculations [23,24] and renormalization group calculations [14,15] up to constant factors. In the NFL regime, since the enhanced single-particle damping of the same order as the thermal excitation energy T breaks the quasi-particle picture, a perturbative expansion up to finite order in terms of the interaction strength is insufficient for the description of the low-energy properties. However, the above exact results show that contributions from higher-order diagrams including the vertex corrections (the second terms of the right-hand side of eqs. (13) and (14)) are negligible for model (1). This observation implies that the classical theory for ferromagnetic fluctuation developed many years ago gives a good approximation for our system [23,24].

In contrast to the normal self-energy, the anomalous self-energy enjoys significant contributions from the vertex corrections, because, as seen from eq. (9), they are not restricted to low-energy regions. These vertex corrections enhance the dynamical retardation effect of the pairing interaction. To examine to what extent this effect affects the transition temperature T_c, we solve the gap equation (9) numerically. For this purpose, as is usually done, we recast eq. (9) into the eigenvalue problem, $\rho(T)\Delta_l(\varepsilon_n) = \sum_m K_{mn}\Delta_l(\varepsilon_m)$. T_c is determined by the temperature at which $\rho(T_c) = 1$. We consider only the p-wave pairing, because it gives the highest T_c of our model. The superconducting T_c calculated as a function of the parameter t, which is the measure of ferromagnetic criticality, is shown in Fig. 2 for a particular set of parameters. Since our analysis is exact up to leading order in $q_c/(2k_F) = x_c$, the errors of the computed T_c are of the order x_cT_c. For comparison, we also show the transition temperatures calculated by neglecting the vertex corrections, T_{c0}, in the same figure. The asymptotically exact T_c is *always higher than* T_{c0}, even if we take into account the errors of the order x_cT_c. The maximum ratio of T_c to T_{c0} is approximately ~ 3. It is noted that in the vicinity of the FC ($t \sim 0$), the transition temperature decreases as t approaches zero. This tendency is similar to the results obtained from the conventional Eliashberg equations [8]. According to Roussev and Millis [8], T_c does not vanish at $t = 0$ within the Eliashberg formalism. Although it is rather difficult to compute T_c at $t = 0$ with numerically reliable accuracy from eqs. (8) and (9), we speculate that it is non-vanishing, since the vertrex corrections always increases T_c. The above results indicate that model (1) is a rare example of a purely repulsive electron system that has been *exactly* proved to undergo the superconducting transition [20].

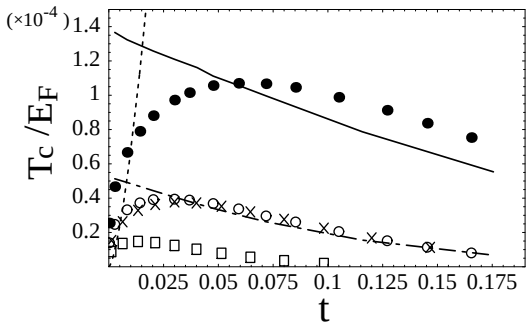

FIG. 2. T_c/E_F versus t for $x_c = 0.3$ (•) and $x_c = 0.2$ (×). T_{c0}/E_F versus t for $x_c = 0.3$ (○) and $x_c = 0.2$ (□). The broken line indicates the crossover temperature T^* which distinguishes the NFL state (high temperature region) and the FL state (low temperature region). The solid and dashed-broken lines indicate $T_c \times 0.1/E_F$ and $T_{c0} \times 0.1/E_F$ for $x_c = 0.3$ in the Ising case, respectively.

Next we consider the case with the Ising-like anisotropy of the exchange interaction. To simplify the calculation, we assume that the Ising anisotropy of spin degrees of freedom is so strong that the interaction part of the Hamiltonian (1) is replaced with $-\sum_q V(q)S^z(q)S^z(-q)$. In this case, the asymptotically exact T_c is obtained from eqs. (8) and (9) with only one modification that the factor g in (8) is replaced with $g = 1$. The transition temperatures computed for this Ising case are shown in Fig. 2 (solid line). The transition temperature without vertex corrections, T_{c0}, are also shown in the figure (dashed-broken line). T_c takes a maximum value at the ferromagnetic critical point $t = 0$ as in the case without vertex corrections [6,8,9]. This is due to the absence of the transverse spin fluctuation which causes the pair-breaking effect, and suppresses T_c for small t in the SU(2) symmetric case. We would like to note that in the case with the strong Ising anisotropy, the loop cancellation theorem holds even in the ferromagnetic metallic state. We can calculate the asymptotically exact T_c in the ferromagnetic state by using eqs. (6) and (7) with $g = 1$, which may be relevant to ferromagnetic superconductors such as UGe_2, URhGe, and $ZrZn_2$.

In conclusion, we have obtained the asymptotically exact solution for the transition temperature of p-wave SC near FC. We have shown that vertex corrections neglected in the conventional Eliashberg equation enhance the dynamical retardation effect of the pairing interaction, increasing the transition temperature significantly.

The author would like to thank K. Yamada and H. Ikeda for valuable discussions. Numerical computations were partly performed at the Yukawa Institute Computer Facility. This work was supported by a Grant-in-Aid from the Ministry of Education, Science, Sports and Culture, Japan.

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